

# Regulator Dependence of the Proposed UV Completion of the Ghost Condensate

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## Abstract

Recently, it was shown that a renormalizable theory of heavy fermions coupled to a light complex boson could generate an effective action for the boson with the properties required to violate Lorentz invariance spontaneously through the mechanism of ghost condensation. However, there was some doubt about whether this result depended on the choice of regulator. In this work, we adopt a non-perturbative, unitary lattice regulator and show that with this regulator the theory does not have the properties necessary to form a ghost condensate. Consequently, the statement that the theory is a UV completion of the Higgs phase of gravity is regulator dependent.

## I. INTRODUCTION

The ghost condensate proposal of [1] has received considerable attention recently [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The condensate is a mechanism for modifying gravity in the infrared. The starting point of the model is a scalar field,  $\phi$ , with a shift symmetry

$$\phi \rightarrow \phi + \alpha \quad (1)$$

such that the effective action for the scalar is of the form  $\mathcal{L} = P(X)$ , where  $X = \partial_\mu \phi \partial^\mu \phi$  (we ignore terms such as  $(\partial^2 \phi)^2$  as they will not be important in our discussion.) Moreover, we assume that  $\phi$  is a ghost, so that  $P(X)$  is of the form shown in Figure 1. The origin,  $\phi = 0$ , is an unstable field configuration in this scenario. The ghost then condenses so that  $(\partial \phi)^2$  has a value near the minimum of  $P$ . It is also possible that there is no ghost at the origin but a non-trivial minimum elsewhere, as shown in Figure 2; in such a theory there would still be a ghost condensate near the minimum of  $P$ . This class of theories is of considerable phenomenological interest because a ghost condensate has equation of state  $w = -1$  and could therefore be relevant for explaining the observed small but non-zero cosmological constant [1].

It is also of interest, however, to understand how the effective action  $\mathcal{L} = P(X)$  could arise as a low energy effective theory of some more familiar UV quantum field theory [9]. Since the scalar field must have a shift symmetry, it is natural to seek a completion in which  $\phi$  is the Goldstone boson of a spontaneously broken  $U(1)$  symmetry. It was shown in [2] that it is impossible, classically, to generate a ghostly low energy effective action for such a Goldstone boson from a high energy theory with standard kinetic terms. However, the authors went on to find a theory in which a quantum correction could change the sign of the kinetic term of the Goldstone boson. In that proposal, all fields start out with standard kinetic terms. However, interactions between  $\phi$  and certain heavy fermions correct the kinetic term of  $\phi$ . It was found that under certain assumptions, these corrections could produce an effective Lagrangian for  $\phi$  of the form shown in Figure 1 at scales much smaller than the fermion mass  $m$ . We do not expect to find an effective Lagrangian of the form shown in Figure 2 because the higher order terms in the expansion of  $P(X)$  are suppressed by powers of the cutoff.

The model described in [2] has some shortcomings. The high energy theory has a Landau pole. Moreover, in dimensional regularization it was found that to change the sign of the

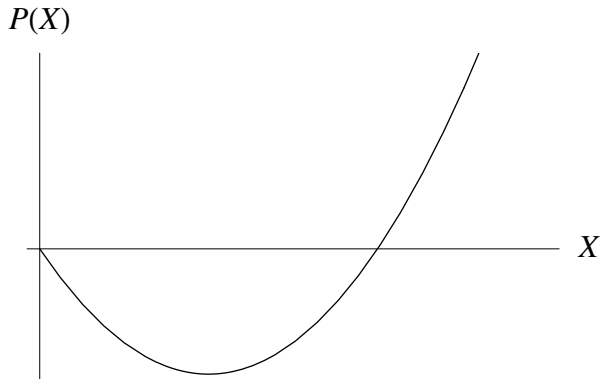


FIG. 1: One possible form for  $P(X)$ .

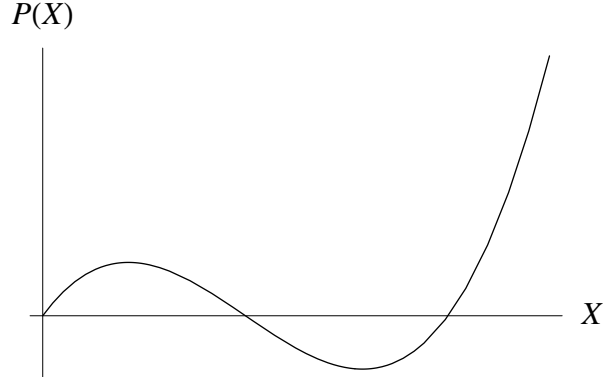


FIG. 2: Another possible form for  $P(X)$ , with no ghost at the origin.

bosonic kinetic term, the mass of the fermions has to be close to the Landau pole. This circumstance may cause some concern that the calculation could be regulator dependent. To alleviate these concerns, the authors demonstrated that their conclusion holds in a large class of momentum-dependent regulators, provided that the fermion masses were taken to be of order of the regulator. These regulators, however, violate unitarity, so again it is not clear to what extent the sign of the kinetic term is a well defined quantity.

In this paper, we re-examine the theory presented in [2] using a lattice regulator. This regulator is non-perturbatively valid and preserves unitarity. We will see that there is never a ghost when the theory is regulated in this way. As a consequence, it seems that the conclusions of [2] are regulator dependent.

## II. COMPUTATION

We begin by describing the theory we will be working with in more detail. The candidate ghost field,  $\phi$ , must have a shift symmetry so it is natural to suppose that it is a Goldstone boson associated with the breaking of some  $U(1)$  symmetry. Hence, following [2], we choose as the bosonic part of the Lagrangian the usual spontaneous symmetry breaking Lagrangian for a complex scalar field  $\Phi$ ,

$$\mathcal{L}_b = \partial_\mu \Phi^* \partial^\mu \Phi - \frac{\lambda}{4} (|\Phi|^2 - v^2)^2. \quad (2)$$

The Goldstone boson,  $\phi$ , associated with the spontaneous symmetry breaking is the candidate ghost field. We couple  $\Phi$  to two families of fermions  $\psi_i$ ,  $i = 1, 2$  of charges  $+1$  and  $-1$  respectively. We will assume that there are  $N$  identical fermions in each family, and that each fermion has the same mass  $m$ . The fermions are coupled to  $\Phi$  by a Yukawa term with coupling  $g$ . Hence, the total Lagrangian density is

$$\mathcal{L} = \mathcal{L}_b + \sum_{j=1}^N \left[ \sum_{i=1,2} \left( i \bar{\psi}_i^{(j)} \gamma^\mu \partial_\mu \psi_i^{(j)} - m \bar{\psi}_i^{(j)} \psi_i^{(j)} \right) - g \Phi \bar{\psi}_2^{(j)} \psi_1^{(j)} - g \Phi^* \bar{\psi}_1^{(j)} \psi_2^{(j)} \right]. \quad (3)$$

The low energy effective action for  $\Phi$  is obtained by integrating the fermions out. The effective action can be written

$$\mathcal{L}_{eff} = \Phi^* G(-\partial^2) \Phi - V(|\Phi|) \quad (4)$$

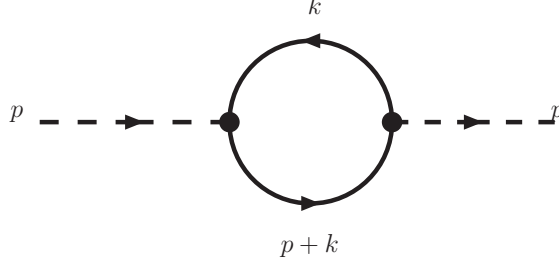


FIG. 3: The relevant Feynman graph. Dashed lines represent the boson while full lines are the fermions.

where

$$G(p^2) = p^2 + g^2 N f(p^2). \quad (5)$$

The function  $f(p^2)$  describes the effects of the quantum corrections to the bosonic kinetic term. If  $G(p^2) < 0$  for some range of  $p^2$ , then the theory can have a ghost. This can only happen if  $g^2 N f(p^2)$  is negative and larger than the tree level term  $p^2$ . Since this signals a breakdown in perturbation theory, we work in the large  $N$  limit with  $g^2 N$  fixed to maintain control over the calculation.

Let us now move on to compute  $f(p^2)$ . To do so, we must evaluate the Feynman graph shown in Figure 3. After Wick rotating both momenta into Euclidean space, we find

$$f(p^2) = -4 \int \frac{d^4 k}{(2\pi)^4} \frac{m^2 - k \cdot (p + k)}{(k^2 + m^2)((p + k)^2 + m^2)}. \quad (6)$$

This expression is divergent and requires regulation. We choose a lattice regulator with lattice spacing  $a$ . Since we are working in the large  $N$  limit, the phenomenon of fermion doubling [13] will not pose a problem. Therefore, we will use naive lattice fermions. The (Euclidean) fermion propagator is given by [13]

$$G(p) = a \frac{-i \sum_{\mu} \gamma_{\mu} \sin(p_{\mu} a) + m a}{\sum_{\mu} \sin^2 p_{\mu} a + m^2 a^2} \quad (7)$$

where  $\gamma_{\mu}$  are Euclidean gamma matrices. On this lattice, momentum components lie in the first Brillouin zone, so  $-\pi < p_{\mu} a < \pi$ . The regulated Feynman graph (fig. 3) is

$$f(p^2) = -4 \int_{\mathcal{B}} \frac{d^4 k}{(2\pi)^4} a^2 \frac{m^2 a^2 - \sum_{\mu} \sin a k_{\mu} \cdot \sin a (p_{\mu} + k_{\mu})}{\left[ \sum_{\nu} \sin^2 a k_{\nu} + m^2 a^2 \right] \left[ \sum_{\rho} \sin^2 a (p_{\rho} + k_{\rho}) + m^2 a^2 \right]} \quad (8)$$

where the integral is over the Brillouin zone  $\mathcal{B}$ . Note that as  $a \rightarrow 0$ , the regulated expression Eq. (8) reduces to the continuum expression Eq. (6).

In [2], there was a ghost at the origin. Since our goal is to check for potential regulator dependence of this statement, it suffices to extract the order  $p^2$  part of  $f(p^2)$ . Thus, we expand Eq. (8) in  $p_{\mu}$  and extract the second order term. We find

$$f(p^2) \simeq -4a^2 \sum_{\mu} p_{\mu} p_{\mu} a^2 \int_{\mathcal{B}} \frac{d^4 k}{(2\pi)^4} \frac{a^2}{(m^2 a^2 + \sum_{\nu} \sin^2 k_{\nu} a)^2} \left[ -\frac{m^2 a^2 - \sum_{\nu} \sin^2 k_{\nu} a}{m^2 a^2 + \sum_{\nu} \sin^2 k_{\nu} a} \cos 2k_{\mu} a \right. \\ \left. + \frac{1}{2} \sin^2 k_{\mu} a + \frac{1}{2} \frac{\sin^2 2k_{\mu} a}{m^2 a^2 + \sum_{\nu} \sin^2 k_{\nu} a} + \frac{m^2 a^2 - \sum_{\nu} \sin^2 k_{\nu} a}{(m^2 a^2 + \sum_{\nu} \sin^2 k_{\nu} a)^2} \sin^2 2k_{\mu} a \right]. \quad (9)$$

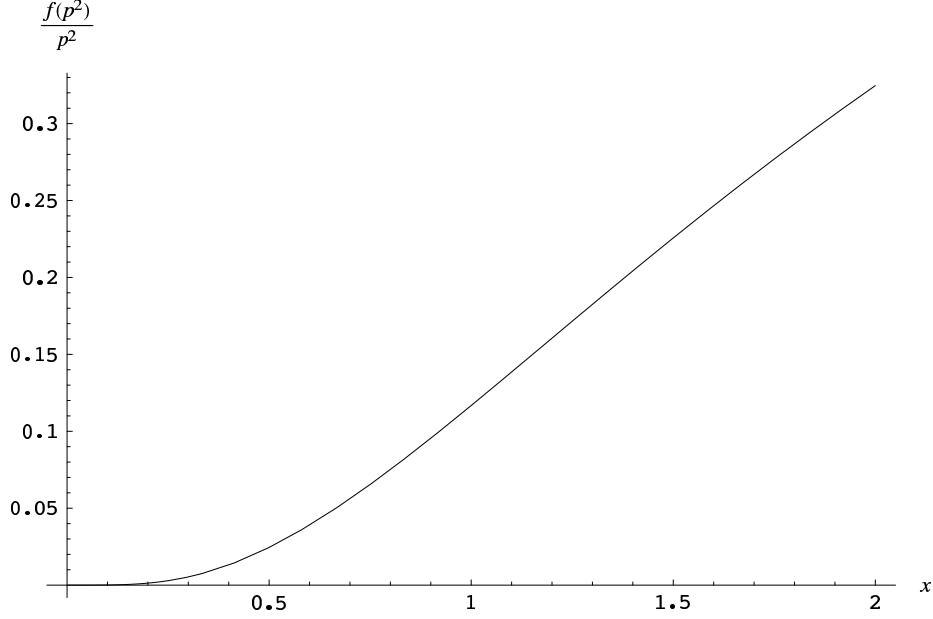


FIG. 4: The quantum correction,  $f(p^2)$  as a function of  $x = 1/ma$ . We have shown  $f(p^2)/p^2$  for clarity.

Now, in [2], the sign of the kinetic term was altered if the fermion mass was taken to be at least of order of the cutoff. For fermion masses large compared to the cutoff, analytic results were obtained demonstrating the presence of a ghost. In our case, we can obtain an analytic result when  $ma \gg 1$ . In this limit, the coefficient of  $p^2$  induced by the quantum correction is given by

$$f(p^2) = -4a^2 \left( \frac{1}{m^2 a^2} \right)^2 \int_{\mathcal{B}} \frac{d^4 k}{(2\pi)^4} \sum_{\mu} \left[ \frac{1}{2} p_{\mu} p_{\mu} a^2 \sin^2 k_{\mu} a - p_{\mu} p_{\mu} a^2 \cos 2k_{\mu} a \right] \quad (10)$$

$$= -4a^2 \left( \frac{1}{m^2 a^2} \right)^2 \frac{p^2}{4a^2}. \quad (11)$$

Rotating back into Euclidean space, we find

$$G(p^2) \simeq p^2 + g^2 N \left( \frac{1}{ma} \right)^4 p^2. \quad (12)$$

Clearly, this quantity never becomes negative, so the sign of the kinetic term does not change in this theory, at least when  $ma \gg 1$ .

To check for a sign change away from this limit, we have numerically integrated Eq. (9) to find the coefficient of  $p^2$  induced by quantum corrections, as a function of  $x = 1/(ma)$ . The result is shown in Figure 4 for  $0 \leq x \leq 2$ . Evidently,  $f(p^2)/p^2$  is never negative, so there can be no change in the sign. For large  $x$ , the fermion mass is much smaller than the cutoff so we need not worry about regulator dependence; therefore, we know from the results of [2] that there is no ghost in the region  $x > 2$ . This completes our demonstration that the sign of the kinetic term is always positive if the theory is regulated on a spacetime lattice.

### III. CONCLUSIONS

We have examined the proposed high energy completion of the ghost condensate [2]. Using a lattice regulator, which is valid without invoking perturbation theory, and which is unitary, we have shown that this theory does not have a ghostly low energy effective action. The effect noted in [2], which involved changing the sign of the kinetic term for a scalar  $\phi$ , appears to be a regulator dependent phenomenon. Thus, the search for a UV completion for the ghost condensate must continue.

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